Reflection and Ray Model of Light - Lesson 3

Concave Mirrors

Image Characteristics for Concave Mirrors

Previously in Lesson 3, ray diagrams were constructed in order to determine the general location, size, orientation, and type of image formed by concave mirrors. Perhaps you noticed that there is a definite relationship between the image characteristics and the location where an object placed in front of a concave mirror. The purpose of this portion of the lesson is to summarize these object-image relationships - to practice the L•O•S•T art of image description. We wish to describe the characteristics of the image for any given object location. The O of L•O•S•T represents the relative location. The L of L•O•S•T represents the orientation (either upright or inverted). The S of L•O•S•T represents the relative size (either magnified, reduced or the same size as the object). And the T of L•O•S•T represents the type of image (either real or virtual). The best means of summarizing this relationship between object location and image characteristics is to divide the possible object locations into five general areas or points:

- Case 1: the object is located beyond the center of curvature (C)
- Case 2: the object is located at the center of curvature (C)
- Case 3: the object is located between the center of curvature (C) and the focal point (F)
- Case 4: the object is located at the focal point (F)
- Case 5: the object is located at the focal point (F)

Case 1: The object is located beyond C

When the object is located at a location beyond the center of curvature, the image will always be located somewhere in between the center of curvature and the focal point. Regardless of exactly where the object is located, the image will be located in the specified region. In this case, the image will be an inverted image. That is to say, if the object is right side up, then the image is upside down. In this case, the image is reduced in size; in other words, the image dimensions are smaller than the object dimensions. If the object is a six-foot tall person, then the image is less than six feet tall. Earlier in Lesson 2, the term magnification was introduced; the magnification is the ratio of the height of the image to the height of the object. In this case, the absolute value of the magnification is less than 1. Finally, the image is a real image. Light rays actually converge at the image location. If a sheet of paper were placed at the image location, the actual replica of the object would appear projected upon the sheet of paper.

Case 2: The object is located at C

When the object is located at the center of curvature, the image will also be located at the center of curvature. In this case, the image will be inverted (i.e., a right side up object results in an upside-down image). The image dimensions are equal to the object dimensions. A six-foot tall person would have an image that is six feet tall; the absolute value of the magnification is equal to 1. Finally, the image is a real image. Light rays actually converge at the image location. As such, the image of the object could be projected upon a sheet of paper.

Case 3: The object is located between C and F

When the object is located in front of the center of curvature, the image will be located beyond the center of curvature. Regardless of exactly where the object is located between C and F, the image will be located somewhere beyond the center of curvature. In this case, the image will be inverted (i.e., a right side up object results in an upside-down image). The image dimensions are larger than the object dimensions. A six-foot tall person would have an image that is larger than six feet tall; the absolute value of the magnification is greater than 1. Finally, the image is a real image. Light rays actually converge at the image location. As such, the image of the object could be projected upon a sheet of paper.

Case 4: The object is located at F

When the object is located at the focal point, no image is formed. As discussed earlier in Lesson 3, light rays from the same point on the object will reflect off the mirror and neither converge nor diverge. After reflecting, the light rays are traveling parallel to each other and do not result in the
formation of an image.

Case 5: The object is located in front of F

When the object is located at a location beyond the focal point, the image will always be located somewhere on the opposite side of the mirror. Regardless of exactly where in front of F the object is located, the image will always be located behind the mirror. In this case, the image will be an **upright image**. That is to say, if the object is right side up, then the image will also be right side up. In this case, the image is **magnified**; in other words, the image dimensions are greater than the object dimensions. A six-foot tall person would have an image that is larger than six feet tall; the magnification is greater than 1. Finally, the image is a virtual image. Light rays from the same point on the object reflect off the mirror and diverge upon reflection. For this reason, the image location can only be found by extending the reflected rays backwards beyond the mirror. The point of their intersection is the virtual image location. It would appear to any observer as though light from the object were diverging from this location. Any attempt to project such an image upon a sheet of paper would fail since light does not actually pass through the image location.

It might be noted from the above descriptions that there is a relationship between the object distance and object size and the image distance and image size. Starting from a large value, as the object distance decreases (i.e., the object is moved closer to the mirror), the image distance increases; meanwhile, the image height increases. At the center of curvature, the object distance equals the image distance and the object height equals the image height. As the object distance approaches one focal length, the image distance and image height approaches infinity. Finally, when the object distance is equal to exactly one focal length, there is no image. Then altering the object distance to values less than one focal length produces images that are upright, virtual and located on the opposite side of the mirror. Finally, if the object distance approaches 0, the image distance approaches 0 and the image height ultimately becomes equal to the object height. These patterns are depicted in the diagram below. Nine different object locations are drawn and labeled with a number; the corresponding image locations are drawn in blue and labeled with the identical number.

**Check Your Understanding**

1. Compare and contrast the images formed by concave and plane mirrors.

2. Identify the means by which you can use a concave and/or a plane mirror to form a real image.

3. Identify the means by which you can use a concave and/or a plane mirror to form a virtual image.

4. Identify the means by which you can use a concave and/or a plane mirror to produce an upright image.
5. Identify the means by which you can use a concave and/or a plane mirror to produce an inverted image.

6. Are all real images larger than the object?

7. The famous Chinese magician, Foo Ling Yu, conducts a classic magic trick utilizing a concave mirror with a focal length of 1.6 m. Foo Ling Yu is able to use the mirror in such a manner as to produce an image of a light bulb at the same location and of the same size as the actual light bulb itself. Use complete sentences to explain how Foo is able to accomplish this magic trick. Be specific about the light bulb location.
Ray Diagrams - Concave Mirrors

Reflection and Ray Model of Light - Lesson 3

Concave Mirrors

The theme of this unit has been that we see an object because light from the object travels to our eyes as we sight along a line at the object. Similarly, we see an image of an object because light from the object reflects off a mirror and travels to our eyes as we sight at the image location of the object. From these two basic premises, we have defined the image location as the location in space where light appears to diverge from. Ray diagrams have been a valuable tool for determining the path taken by light from the object to the mirror to our eyes. In this section of Lesson 3, we will investigate the method for drawing ray diagrams for objects placed at various locations in front of a concave mirror. To draw these diagrams, we will have to recall the two rules of reflection for concave mirrors:

• Any incident ray traveling parallel to the principal axis on the way to the mirror will pass through the focal point upon reflection.
• Any incident ray passing through the focal point on the way to the mirror will travel parallel to the principal axis upon reflection.

Earlier in this lesson, the following diagram was shown to illustrate the path of light from an object to mirror to an eye.

![Ray Diagrams - Concave Mirrors](image_url)

In this diagram five incident rays are drawn along with their corresponding reflected rays. Each ray intersects at the image location and then diverges to the eye of an observer. Every observer would observe the same image location and every light ray would follow the law of reflection. Yet only two of these rays would be needed to determine the image location since it only requires two rays to find the intersection point. Of the five incident rays drawn, two of them correspond to the incident rays described by our two rules of reflection for concave mirrors. Because they are the easiest and most predictable pair of rays to draw, these will be the two rays used through the remainder of this lesson.

Step-by-Step Method for Drawing Ray Diagrams

The method for drawing ray diagrams for concave mirror is described below. The method is applied to the task of drawing a ray diagram for an object located beyond the center of curvature (C) of a concave mirror. Yet the same method works for drawing a ray diagram for any object location.

1. Pick a point on the top of the object and draw two incident rays traveling towards the mirror.

   Using a straight edge, accurately draw one ray so that it passes exactly through the focal point on the way to the mirror. Draw the second ray such that it travels exactly parallel to the principal axis. Place arrowheads upon the rays to indicate their direction of travel.

2. Once these incident rays strike the mirror, reflect them according to the two rules of reflection for concave mirrors.

   The ray that passes through the focal point on the way to the mirror will reflect and travel parallel to the principal axis. Use a straight edge to accurately draw its path. The ray that traveled parallel to the principal axis on the way to the mirror will reflect and travel through the focal point. Place arrowheads upon the rays to indicate their...
direction of travel. Extend the rays past their point of intersection.

3. Mark the image of the top of the object.

The image point of the top of the object is the point where the two reflected rays intersect. If you were to draw a third pair of incident and reflected rays, then the third reflected ray would also pass through this point. This is merely the point where all light from the top of the object would intersect upon reflecting off the mirror. Of course, the rest of the object has an image as well and it can be found by applying the same three steps to another chosen point. (See note below.)

4. Repeat the process for the bottom of the object.

The goal of a ray diagram is to determine the location, size, orientation, and type of image that is formed by the concave mirror. Typically, this requires determining where the image of the upper and lower extreme of the object is located and then tracing the entire image. After completing the first three steps, only the image location of the top extreme of the object has been found. Thus, the process must be repeated for the point on the bottom of the object. If the bottom of the object lies upon the principal axis (as it does in this example), then the image of this point will also lie upon the principal axis and be the same distance from the mirror as the image of the top of the object. At this point the entire image can be filled in.

Some students have difficulty understanding how the entire image of an object can be deduced once a single point on the image has been determined. If the object is a vertically aligned object (such as the arrow object used in the example below), then the process is easy. The image is merely a vertical line. In theory, it would be necessary to pick each point on the object and draw a separate ray diagram to determine the location of the image of that point. That would require a lot of ray diagrams as illustrated below.

Fortunately, a shortcut exists. If the object is a vertical line, then the image is also a vertical line. For our purposes, we will only deal with the simpler situations in which the object is a vertical line that has its bottom located upon the principal axis. For such simplified situations, the image is a vertical line with the lower extremity located upon the principal axis.

The ray diagram above illustrates that when the object is located at a position beyond the center of curvature, the image is located at a position between the center of curvature and the focal point. Furthermore, the image is inverted, reduced in size (smaller than the object), and real. This is the type of information that we wish to obtain from a ray diagram. These characteristics of the image will be discussed in more detail in the next section of Lesson 3.

Once the method of drawing ray diagrams is practiced a couple of times, it becomes as natural as breathing. Each diagram yields specific information about the image. The two diagrams below show how to determine image location, size, orientation and type for situations in which the object is located at the center of curvature and when the object is located...
between the center of curvature and the focal point.

![Animation](http://www.physicsclassroom.com/Class/refln/U13L3d.cfm)

It should be noted that the process of constructing a ray diagram is the same regardless of where the object is located. While the result of the ray diagram (image location, size, orientation, and type) is different, the two rules of reflection are applied in order to determine the location where all reflected rays appear to diverge from (which for real images, is also the location where the reflected rays intersect).

In the three cases described above - the case of the object being located beyond C, the case of the object being located at C, and the case of the object being located between C and F - light rays are converging to a point after reflecting off the mirror. In such cases, a real image is formed. As discussed previously, a real image is formed whenever reflected light passes through the image location. While plane mirrors always produce virtual images, concave mirrors are capable of producing both real and virtual images. As shown above, real images are produced when the object is located a distance greater than one focal length from the mirror. A virtual image is formed if the object is located less than one focal length from the concave mirror. To see why this is so, a ray diagram can be used.

**Ray Diagram for the Formation of a Virtual Image**

A ray diagram for the case in which the object is located in front of the focal point is shown in the diagram at the right. Observe that in this case the light rays diverge after reflecting off the mirror. When light rays diverge after reflection, a virtual image is formed. As was done with plane mirrors, the image location can be found by tracing all reflected rays backwards until they intersect. For every observer, the reflected rays would seem to be diverging from this point. Thus, the point of intersection of the extended reflected rays is the image point. Since light does not actually pass through this point (light never travels behind the mirror), the image is referred to as a virtual image. Observe that when the object in located in front of the focal point, its image is an upright and enlarged image that is located on the other side of the mirror. In fact, one generalization that can be made about all virtual images produced by mirrors (both plane and curved) is that they are always upright and always located on the other side of the mirror.

**Ray Diagram for an Object Located at the Focal Point**

Thus far we have seen via ray diagrams that a real image is produced when an object is located more than one focal length from a concave mirror; and a virtual image is formed when an object is located less than one focal length from a concave mirror (i.e., in front of F). But what happens when the object is located at F? That is, what type of image is formed when the object is located exactly one focal length from a concave mirror? Of course a ray diagram is always one tool to help find the answer to such a question. However, when a ray diagram is used for this case, an immediate difficulty is encountered. The incident ray that begins from the top extremity of the object and passes through the focal point does not meet the mirror. Thus, a different incident ray must be used in order to determine the intersection point of all reflected rays. Any incident light ray would work as long as it meets up with the mirror. Recall that the only reason that we have used the two we have is that they can be conveniently and easily drawn. The diagram below shows two incident rays and their corresponding reflected rays.
For the case of the object located at the focal point (F), the light rays neither converge nor diverge after reflecting off the mirror. As shown in the diagram above, the reflected rays are traveling parallel to each other. Subsequently, the light rays will not converge on the object's side of the mirror to form a real image; nor can they be extended backwards on the opposite side of the mirror to intersect to form a virtual image. So how should the results of the ray diagram be interpreted? The answer: there is no image!! Surprisingly, when the object is located at the focal point, there is no location in space at which an observer can sight from which all the reflected rays appear to be diverging. An image is not formed when the object is located at the focal point of a concave mirror.

**Check Your Understanding**

The diagram below shows two light rays emanating from the top of the object and incident towards the mirror. Describe how the reflected rays for these light rays can be drawn without actually using a protractor and the law of reflection.
Reflection of Light and Image Formation

Light always follows the **law of reflection**, whether the reflection occurs off a curved surface or off a flat surface. The task of determining the direction in which an incident light ray would reflect involves determining the normal to the surface at the point of incidence. For a concave mirror, the normal at the point of incidence on the mirror surface is a line that extends through the **center of curvature**. Once the normal is drawn, the **angle of incidence** can be measured and the reflected ray can be drawn with the same angle. This process is illustrated with two separate incident rays in the diagram at the right.

Lesson 2 discussed the **formation of images** by plane mirrors. In Lesson 2, it was emphasized the image location is the location where reflected light appears to diverge from. For plane mirrors, **virtual images** are formed. Light does not actually pass through the virtual image location; it only appears to an observer as though the light is emanating from the virtual image location. In this lesson we will begin to see that concave mirrors are capable of producing **real images** (as well as virtual images). When a real image is formed, it still appears to an observer as though light is diverging from the real image location. Only in the case of a real image, light is actually passing through the image location.

Suppose that a light bulb is placed in front of a concave mirror at a location somewhere **behind** the center of curvature (C). The light bulb will emit light in a variety of directions, some of which will strike the mirror. Each individual ray of light that strikes the mirror will reflect according to the law of reflection. Upon reflecting, the light will converge at a point. At the point where the light from the object converges, a replica, likeness or reproduction of the actual object is created. This replica is known as the **image**. Once the reflected light rays reach the image location, they begin to diverge. The point where all the reflected light rays converge is known as the image point. Not only is it the point where light rays converge, it is also the point where reflected light rays appear to an observer to be diverging from. Regardless of the observer’s location, the observer will see a ray of light passing through the real image location. To view the image, the observer must line her sight up with the image location in order to see the image via the reflected light ray. The diagram below depicts several rays from the object reflecting from the mirror and converging at the image location. The reflected light rays then begin to diverge, with each one being capable of assisting an individual in viewing the image of the object.

If the light bulb is located at a different location, the same principles apply. The image location is the location where reflected light appears to diverge from. By determining the path that light from the bulb takes after reflecting from the mirror, the image location can be identified. The diagram below depicts this concept.
You might notice that while the same principle applies for determining the image location, a different result is obtained. When the object is located beyond the center of curvature (C), the image is located between the center of curvature (C) and the focal point (F). On the other hand, when the object is located between the center of curvature (C) and the focal point (F), the image is located beyond the center of curvature (C). Unlike plane mirrors, the object distance is not necessarily equal to the image distance. The actual relationship between object distance and image distance is dependent upon the location of the object. These ideas will be discussed in more detail later in this lesson.
**Spherical Aberration**

Aberration - a departure from the expected or proper course. (Webster's Dictionary)

Spherical mirrors have an aberration. There is an intrinsic defect with any mirror that takes on the shape of a sphere. This defect prohibits the mirror from focusing all the incident light from the same location on an object to a precise point. The defect is most noticeable for light rays striking the outer edges of the mirror. Rays that strike the outer edges of the mirror fail to focus in the same precise location as light rays that strike the inner portions of the mirror. While light rays originating at the same location on an object reflect off the mirror and focus to a point, any light rays striking the edges of the mirror fail to focus at that same point. The result is that the images of objects as seen in spherical mirrors are often blurry.

The diagram below shows six incident rays traveling parallel to the principal axis and reflecting off a concave mirror. The six corresponding reflected rays are also shown. In the diagram we can observe a departure from the expected or proper course; there is an aberration. The two incident rays that strike the outer edges (top and bottom) of the concave mirror fail to pass through the focal point. This is a departure from the expected or proper course.

This problem is not limited to light that is incident upon the mirror and traveling parallel to the principal axis. Any incident ray that strikes the outer edges of the mirror is subject to this departure from the expected or proper course. A common Physics demonstration utilizes a large demonstration mirror and a candle. The image of the candle is first projected upon a screen and focused as closely as possible. While the image is certainly discernible, it is slightly blurry. Then a cover is placed over the outer edges of the large demonstration mirror. The result is that the image suddenly becomes more clear and focused. When the problematic portion of the mirror is covered so that it can no longer focus (or mis-focus) light, the image appears more focused.

Spherical aberration is most commonly corrected by use of a mirror with a different shape. Usually, a parabolic mirror is substituted for a spherical mirror. The outer edges of a parabolic mirror have a significantly different shape than that of a spherical mirror. Parabolic mirrors create sharp, clear images that lack the blurriness which is common to those images produced by spherical mirrors.
Thus far in this unit, our focus has been the reflection of light off flat surfaces and the formation of images by plane mirrors. In Lessons 3 and 4 we will turn our attention to the topic of curved mirrors, and specifically curved mirrors that have a spherical shape. Such mirrors are called spherical mirrors. The two types of spherical mirrors are shown in the diagram on the right. Spherical mirrors can be thought of as a portion of a sphere that was sliced away and then silvered on one of the sides to form a reflecting surface. Concave mirrors were silvered on the inside of the sphere and convex mirrors were silvered on the outside of the sphere. In Lesson 3 we will focus on concave mirrors and in Lesson 4 we will focus on convex mirrors.

Beginning a study of spherical mirrors demands that you first become acquainted with some terminology that will be periodically used. The internalized understanding of the following terms will be essential during Lessons 3 and 4.

- **Principal axis**
- **Center of Curvature**
- **Vertex**
- **Focal Point**
- **Radius of Curvature**
- **Focal Length**

If a concave mirror were thought of as being a slice of a sphere, then there would be a line passing through the center of the sphere and attaching to the mirror in the exact center of the mirror. This line is known as the principal axis. The point in the center of the sphere from which the mirror was sliced is known as the center of curvature and is denoted by the letter C in the diagram below. The point on the mirror's surface where the principal axis meets the mirror is known as the vertex and is denoted by the letter A in the diagram below. The vertex is the geometric center of the mirror. Midway between the vertex and the center of curvature is a point known as the focal point; the focal point is denoted by the letter F in the diagram below. The distance from the vertex to the center of curvature is known as the radius of curvature (represented by R). The radius of curvature is the radius of the sphere from which the mirror was cut. Finally, the distance from the mirror to the focal point is known as the focal length (represented by f). Since the focal point is the midpoint of the line segment adjoining the vertex and the center of curvature, the focal length would be one-half the radius of curvature.

The focal point is the point in space at which light incident towards the mirror and traveling parallel to the principal axis will meet after reflection. The diagram at the right depicts this principle. In fact, if some light from the sun were collected by a concave mirror, then it would converge at the focal point. Because the sun is such a large distance from the Earth, any light rays from the sun that strike the mirror will essentially be traveling parallel to the principal axis. As such, this light should reflect and pass through the focal point. A common Physics demonstration involves using a large demonstration mirror to set a pencil aflame in a matter of seconds. In the demonstration, the pencil is placed at the focal point and the concave mirror is pointed upwards towards the sun. Whatever rays of light from the sun that hit the mirror are focused at the point where the pencil is located. To the surprise of many, the heat is sufficient to ignite the pencil. Wow!

As we proceed through Lesson 3, we will observe the images formed by concave mirrors. Depending on the object location, the image could be enlarged or reduced in size or even the same size as the object; the image could be inverted or upright; and the image will be located in a specific region along the principal axis. To understand these relationships between object and image, you may need to review the vocabulary terms described on this page.
Check Your Understanding

1. The surface of a concave mirror is pointed towards the sun. Light from the sun hits the mirror and converges to a point. How far is this converging point from the mirror's surface if the radius of curvature (R) of the mirror is 150 cm?

2. It’s the early stages of a concave mirror lab. Your teacher hands your lab group a concave mirror and asks you to find the focal point. What procedure would you use to do this?
The Mirror Equation

Ray diagrams can be used to determine the image location, size, orientation and type of image formed of objects when placed at a given location in front of a concave mirror. The use of these diagrams was demonstrated earlier in Lesson 3. Ray diagrams provide useful information about object-image relationships, yet fail to provide the information in a quantitative form. While a ray diagram may help one determine the approximate location and size of the image, it will not provide numerical information about image distance and object size. To obtain this type of numerical information, it is necessary to use the **Mirror Equation** and the **Magnification Equation**. The mirror equation expresses the quantitative relationship between the object distance ($d_o$), the image distance ($d_i$), and the focal length ($f$). The equation is stated as follows:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

The magnification equation relates the ratio of the image distance and object distance to the ratio of the image height ($h_i$) and object height ($h_o$). The magnification equation is stated as follows:

$$M = \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

These two equations can be combined to yield information about the image distance and image height if the object distance, object height, and focal length are known.

As a demonstration of the effectiveness of the mirror equation and magnification equation, consider the following example problem and its solution.

**Example Problem #1**

A 4.00-cm tall light bulb is placed a distance of 45.7 cm from a concave mirror having a focal length of 15.2 cm. Determine the image distance and the image size.

Like all problems in physics, begin by identifying the known information.

$$h_o = 4.0 \text{ cm} \quad \quad \quad d_o = 45.7 \text{ cm} \quad \quad \quad f = 15.2 \text{ cm}$$

Next identify the unknown quantities that you wish to solve for.

$$d_i = ??? \quad \quad \quad h_i = ??$$

To determine the image distance, the mirror equation must be used. The following lines represent the solution to the image distance; substitutions and algebraic steps are shown.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{(15.2 \text{ cm})} = \frac{1}{(45.7 \text{ cm})} + \frac{1}{d_i}$$

$$0.0658 \text{ cm}^{-1} = 0.0219 \text{ cm}^{-1} + \frac{1}{d_i}$$

$$0.0439 \text{ cm}^{-1} = \frac{1}{d_i}$$

$$d_i = 22.8 \text{ cm}$$

The numerical values in the solution above were rounded when written down, yet un-rounded numbers were used in all calculations. The final answer is rounded to the third significant digit.

To determine the image height, the magnification equation is needed. Since three of the four quantities in the equation (disregarding the $M$) are known, the fourth quantity can be calculated. The solution is shown below.

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$
The negative values for image height indicate that the image is an inverted image. As is often the case in physics, a negative or positive sign in front of the numerical value for a physical quantity represents information about direction. In the case of the image height, a negative value always indicates an inverted image.

From the calculations in this problem it can be concluded that if a 4.00-cm tall object is placed 45.7 cm from a concave mirror having a focal length of 15.2 cm, then the image will be inverted, 1.99-cm tall and located 22.8 cm from the mirror. The results of this calculation agree with the principles discussed earlier in this lesson. In this case, the object is located beyond the center of curvature (which would be two focal lengths from the mirror), and the image is located between the center of curvature and the focal point. This falls into the category of Case 1: The object is located beyond C.

Now let's try a second example problem:

**Example Problem #2**

A 4.0-cm tall light bulb is placed a distance of 8.3 cm from a concave mirror having a focal length of 15.2 cm. (NOTE: this is the same object and the same mirror, only this time the object is placed closer to the mirror.) Determine the image distance and the image size.

Again, begin by the identification of the known information.

\[ h_o = 4.0 \text{ cm} \quad d_o = 8.3 \text{ cm} \quad f = 15.2 \text{ cm} \]

Next identify the unknown quantities that you wish to solve for.

\[ d_i = ??? \quad h_i = ??? \]

To determine the image distance, the mirror equation will have to be used. The following lines represent the solution to the image distance; substitutions and algebraic steps are shown.

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \\
\frac{1}{(15.2 \text{ cm})} = \frac{1}{(8.3 \text{ cm})} + \frac{1}{d_i} \\
0.0658 \text{ cm}^{-1} = 0.120 \text{ cm}^{-1} + \frac{1}{d_i} \\
-0.0547 \text{ cm}^{-1} = \frac{1}{d_i} \\
d_i = -18.3 \text{ cm} 
\]

The numerical values in the solution above were rounded when written down, yet un-rounded numbers were used in all calculations. The final answer is rounded to the third significant digit.

To determine the image height, the magnification equation is needed. Since three of the four quantities in the equation (disregarding the M) are known, the fourth quantity can be calculated. The solution is shown below.

\[
\frac{h_i}{h_o} = - \frac{d_i}{d_o} \\
\frac{h_i}{(4.0 \text{ cm})} = -\frac{(18.2 \text{ cm})}{(8.3 \text{ cm})} \\
h_i = - (4.0 \text{ cm}) \times (-18.2 \text{ cm})/(8.3 \text{ cm}) \\
h_i = 8.8 \text{ cm} 
\]

The negative value for image distance indicates that the image is a virtual image located behind the mirror. Again, a negative or positive sign in front of the numerical value for a physical quantity represents information about direction. In the case of the image distance, a negative value always means behind the mirror. Note also that the image height is a positive value, meaning an upright image. Any image that is upright and located behind the mirror is considered to be a virtual image.

From the calculations in the second example problem it can be concluded that if a 4.0-cm tall object is placed 8.3 cm from a concave mirror having a focal length of 15.2 cm, then the image will be magnified, 8.8-cm tall and located 18.3 cm behind the mirror. The results of this calculation agree with the principles discussed earlier in this lesson. In this case, the object is located in front of the focal point (i.e., the object distance is less than the focal length), and the image is located behind the mirror. This falls into the category of Case 5: The object is located in front of F.
The +/- Sign Conventions

The sign conventions for the given quantities in the mirror equation and magnification equations are as follows:

- f is + if the mirror is a concave mirror
- f is - if the mirror is a convex mirror
- \( d_i \) is + if the image is a real image and located on the object's side of the mirror.
- \( d_i \) is - if the image is a virtual image and located behind the mirror.
- \( h_i \) is + if the image is an upright image (and therefore, also virtual)
- \( h_i \) is - if the image an inverted image (and therefore, also real)

Like many mathematical problems in physics, the skill is only acquired through much personal practice. Perhaps you would like to take some time to try the problems in the Check Your Understanding section below.

Check Your Understanding

1. Determine the image distance and image height for a 5.00-cm tall object placed 45.0 cm from a concave mirror having a focal length of 15.0 cm.

2. Determine the image distance and image height for a 5.00-cm tall object placed 30.0 cm from a concave mirror having a focal length of 15.0 cm.

3. Determine the image distance and image height for a 5.00-cm tall object placed 20.0 cm from a concave mirror having a focal length of 15.0 cm.

4. Determine the image distance and image height for a 5.00-cm tall object placed 10.0 cm from a concave mirror having a focal length of 15.0 cm.

5. A magnified, inverted image is located a distance of 32.0 cm from a concave mirror with a focal length of 12.0 cm. Determine the object distance and tell whether the image is real or virtual.

ZINGER: 6. An inverted image is magnified by 2 when the object is placed 22 cm in front of a concave mirror. Determine the image distance and the focal length of the mirror.
Two Rules of Reflection for Concave Mirrors

Light always reflects according to the law of reflection, regardless of whether the reflection occurs off a flat surface or a curved surface. Using reflection laws allows one to determine the image location for an object. The image location is the location where all reflected light appears to diverge from. Thus to determine this location demands that one merely needs to know how light reflects off a mirror. In the previous section of Lesson 3, the image of an object for a concave mirror was determined by tracing the path of light as it emanated from an object and reflected off a concave mirror. The image was merely that location where all reflected rays intersected. The use of the law of reflection to determine a reflected ray is not an easy task. For each incident ray, a normal line at the point of incidence on a curved surface must be drawn and then the law of reflection must be applied. A simpler method of determining a reflected ray is needed.

The simpler method relies on two rules of reflection for concave mirrors. They are:

1. Any incident ray traveling parallel to the principal axis on the way to the mirror will pass through the focal point upon reflection.
2. Any incident ray passing through the focal point on the way to the mirror will travel parallel to the principal axis upon reflection.

These two rules of reflection are illustrated in the diagram below.

These two rules will greatly simplify the task of determining the image locations for objects placed in front of concave mirrors. In the next section of Lesson 3, these two rules will be applied to determine the location, orientation, size and type of image produced by a concave mirror. As the rules are applied in the construction of ray diagrams, do not forget the fact that the law of reflection holds for each of these rays. It just so happens that when the law of reflection is applied for a ray (either traveling parallel to the principal axis or passing through F) that strikes the mirror at a location near the principal axis, the ray will reflect in close approximation with the above two rules.